Proceedings of the Jangjeon Mathematical Society www.jangjeon.or.kr
22 (2019), No. 3. pp. 401 - 414 http://dx.doi.org/10.17777/pjms2019.22.3.401

RELIABILITY ANALYSIS OF ACYCLIC TRANSMISSION NETWORK BASED ON MINIMAL CUTS USING COPULA IN REPAIR

SONI BISHT AND S.B. SINGH

Abstract. In the present paper, we have considered acyclic transmission network in which number of nodes are capable of receiving or sending a signal to the target nodes. To model the proposed acyclic transmission network, the present study combined the concepts of Markov processes and minimal cuts incorporating copula to find the various reliability measures. The considered network can have four possible states namely operable, partial failure, critical failure and complete failure. The proposed network can be repaired in two different ways. When the network is in critical state it is repaired with general repair, whereas, in complete failure state it is repaired with the help of two different repair rates, namely, general and exponential. The general and exponential distributions have been incorporated with the application of the Gumbel-Hougaard family of copula. Various reliability characteristics such as transition state probabilities, asymptotic behavior, reliability, mean time to failure and sensitivity of the proposed network has been evaluated with the help of minimal cuts coupling with Markov processes using GumbelHougaard copula, supplementary variable techniques and Laplace transforms.
Keywords: Network reliability, Acyclic transmission network, Mean time to failure, Sensitivity, Supplementary variable techniques, Gumbel-Hougaard family of copula.

1. introduction

Networks are the prominent part in many real-world systems such as computer architecture, data communications, software engineering, voice communications, transportations, oil and gas production and electrical power systems (Gertsbakh and Shpungin 2016). A network is a combination of nodes and links which is also known as vertices and edges respectively. A network model is defined by G= (V, E) in which V and E show nodes and edges set respectively. Examples of nodes (vertices) are railway stations, road intersections, mobile routers in mobile communication etc. and links (edges) are railways, roads, wireless paths etc. (Colbourn 1987). Network Reliability is the probability of transferring the information/flows/messages from a source node to a sink node. It is an important concept in the planning, designing, manufacturing, and maintenance of controlling of networks. The network reliability evaluation problem occurs in a wide range of situations including telecommunications, interconnection networks, parallel processing networks and many others. Reliability of networks can be classified on the basis of networks connectivity in three ways. When all terminals are connected to each other, then it is called all-to-all communication reliability or network reliability. If there is a connection from one source to all sink nodes, then it is known as one-to-all communication reliability or broadcast reliability. In case of network connection from the one source node to the one sink node, it is called terminal reliability. Traditionally, network reliability considers binary state networks in which both the components and system can possibly be in two states: completely working or totally failed. However, network reliability analysis in the context of multi-state network is based on performance level not on the connectivity level (Gertsbakh and Shpungin 2016). There are two types of network reliability problems based on flow conservation law i.e. Binary state flow network (BFN)/multi-state flow network (MFN) and Multi-state node network (MNN). In the BFN, the capacity of each arc is either 0 or 1 whereas, in the MFN, the capacity of each arc can be a non-negative integer. The BFN/MFN satisfies the flow conservation law whereas MNN does not. These networks have their own utilization in many real-life problems, like electrical power distribution, transportation networks, cellular telephones and computer networks. For further discussion interested readers can refer to (Levitin 2005, Yeh 2012). The considered acyclic transmission network (ATN) consists of a number of nodes which are capable of receiving and sending a signal via different edges. The proposed network consists of a root node where the signal source is placed, a number of leaf nodes that can only receive a signal and a number of intermediate nodes neither source node nor sink node which are capable of transmitting the received signal to some other nodes. The whole network is in working condition if a signal from the root node is transmitted to the leaf nodes, otherwise, the network fails. An example of the acyclic transmission network is a radio relay station, where a transmitter is located at the root node (position) and receivers are located in the terminal nodes (positions). The aim of this network is to propagate the signals from transmitters to the receivers (Gertsbakh and Shpungin 2016, Yeh 2006). (Levitin 2001, 2003) evaluated the reliability for acyclic transmission networks of multi-state elements with time delays and without time delays from an algorithm based on the extended universal generating function method. In this work it is assumed that the network fails if the signal generated at the source node cannot reach the terminal nodes within a specified time. A Multistatenode acyclic network (MNAN) was first investigated by Malinowski and Preuss in 1996. In this study researchers evaluated the reliability of multistate node of acyclic networks using minimal cuts (MC) based on an algorithm and some simple concepts. (Levitin 2001 and Malinowski and Preuss 1996) also discussed reliability evaluations for MNAN by one of the best-known algorithms and these algorithms are based on UGF technique and branch and bound method. (Levitin 2005) discussed different algorithms and applied UGF method to find the reliability of binary and multi state systems. Reliability analysis of the systems is traditionally done with the help of probability distributions. Usually a single distribution is used in failure/repair analysis. But if two different distributions are to be applied simultaneously in the repair /failure, then one may employ copula to deal with the incorporation. The copula is a function which joins or couples a multivariate distribution function to its one-dimensional marginal distribution functions. Copulas are multivariate distributions functions whose one-dimensional margins are uniform on the interval [0, 1].The copula approach is very natural when any system is repaired/failed via multiple mechanisms (Nelsen 2006). There are some important families of copulas having their own characteristics. The family of Archimedean copulas has been studied by numbers of authors. (Ram and Singh 2008) applied the Gumbel-Hougaard family of copula and determined the availability, MTTF and cost analysis of complex systems under preemptive repeat repair discipline. (Nailwal and Singh 2016) calculated the reliability of cold standby redundant systems with preventive maintenance using Gumbel-Hougaard family of the copula. (Munjal and Singh 2014) considered the complex repairable system consisting of 2-out-of-3: G subsystem (A 2-outof- 3: G system consists of 3 components and works if and only if the total number of working components is at least 2) connected in parallel for finding the reliability characteristics and using Gumbel-Hougaard family of copula. In (Nailwal and Singh 2011), the researchers evaluated the performance and reliability analysis of a complex system having three types of repairs with the application of copula. (Srinivasan and Subramanian 2006) considered standby systems with more than two units and these systems are studied only when either the lifetime or the repair time is exponentially distributed. (Kumar and Singh 2013) discussed the reliability analysis of a complex system having two repairable subsystems viz. A and B connected in series. This study also included a special type of delay, viz. reboots delay and used Gumbel- Hougaard family of copula to obtain various transition state probabilities, reliability, availability, MTTF, cost analysis and sensitivity analysis. (Nailwal and Singh 2012) investigated the reliability characteristics of a complex system having nine subsystems arranged in the form of a matrix in which each row contains three subsystems. The considered system analyzed the different types of power failures which also lead to failure of the system. From the above discussion, it is clear that many researchers have analyzed the reliability of different networks by incorporating the probabilistic evaluations based on inclusion-exclusion, the sum of disjoint products methods and universal generating function (UGF). Researchers have also analyzed the reliability of acyclic transmission network by using UGF method and also discussed many algorithms based on different concepts like minimal cut, Dijkstras and Kruskals algorithm. Further, it is also clear from the above discussion that reliability analysis of the acyclic transmission network using both Markov processes and minimal cuts yet to be studied. Keeping the above facts in view, here in the proposed work we have tried to combine the concepts of Markov processes and minimal cuts to evaluate the reliability characteristics of the acyclic transmission network. In the considered network there are four different possible states, namely operable, partial failure, critical failure and complete failure. The network is said to be in partial failure state, if one of the edges fails but the signal is transmitted to both the sinks. But if any further failure occurs in the network and the signal flows to only one of the sinks then it is said to be in critical state. If there is no flow to any of the sinks then the network is in complete failure state. When the network is in the critical states, then the repair is exponentially distributed whereas when it is in complete failure state then the repair of network follows two different distributions, namely, general and exponential. The proposed acyclic transmission network has been studied to evaluate the reliability characteristics with the application of minimal cuts and Markov processes incorporating the Gumbel-Hougaard family of copula, supplementary variable techniques and Laplace transforms. The reliability measures such as transition state probabilities, asymptotic behavior, MTTF and sensitivity of the network have been obtained. The considered acyclic transmission network and the transition diagram of the network are shown in Figures 1 and 2 respectively

Assumptions:

(i) Initially, the acyclic transmission network is in the good state.

(ii) In the network there are four possible minimal cuts,
viz.{1, 2}{1, 3},{3,5}{2, 4},{1, 2}{2, 3}{3, 5},{1, 3}{2, 3}{2, 4}

(iii) The network has four possible states: good, degraded, critical and completely failed.

(iv) Network has three types of failure: partial, critical and complete failure.

(v) The network is repaired when it is in the critical and complete failure states.

(vi) Transitions from critical states S_6 and S_9 to initial state S_0 follow the general distribution.

(vii)Transitions from the completely failed states S_2, S_4, S_7 and S_{10} to initial state S_0 follow two dierent distributions incorporating Gumbel-Hougaard family of the copula.

Table1: Descriptions of notations used in Transition Diagram	
States	Descriptions
S_0	The state when all the edges of the network are in working condition.
S_1	The state when the edge{1, 3} fails and the network is in degraded state.
S_2	The state when the edges {1, 2} and {1, 3} in the network fail and network is in completely failed state.
S_3	The state when the edge {3, 5} fails and the network is in the degraded state.
S_4	The state when the edges {3, 5} and {2, 4} in the network fail and the network is in the completely failed state.
S_5	The state when the edge {3, 5} fails and the network is in the degraded state.
S_6	The state when the edges {1, 2} and {2,3} in the network fail and the network is in the critical state.
S_7	The state when the edges {1, 2},{2,3} and{3, 5} in the network fail and the network is in the completely failed state.
S_8	The state when the edge {1, 3} fails then the network is in the degraded state.
S_9	The state when the edges {1, 3} and {2, 3} in the network fail and the network is in the critical state.
S_{10}	The state when the edges {1, 3}, {2, 3} and {2, 4}in the network fail and the network is in the completely failed state.
$\lambda_{12}/\lambda_{13}/\lambda_{24}/\lambda_{35}/\lambda_{23}$	The different failure rates corresponding to their edges in the network using possible minimal cuts.
$P_i(t)$	The probability that the network is in Si state at instant time t, i=1 to 10.
$\overline{P}_i(s)$	Laplace transform of $P_i(t)$.
$\phi_1(x)$	Repair rate in states S_6 and S_9 in the network with the elapsed repair time x.
$\phi_2(x)$	Repair rate in states S_2, S_4, S_7 and S_{10} in the network with the elapsed repair time x.
$\varphi(x)$	Coupled repair rate.

Letting $u_1 = e^x$ and $u_1 = \phi_2(x)$ the expression for joint probability (completely failed state S_2,S_4,S_7 and S_{10} to good state S_0) according to Gumbel-Hougaard family of copula is given by

$$\varphi(x) = exp[(x^\theta + (log\phi_2(x))^\theta]^{\frac{1}{\theta}}$$

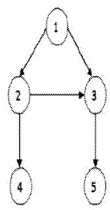

Fig.1: Acyclic Transmission Network

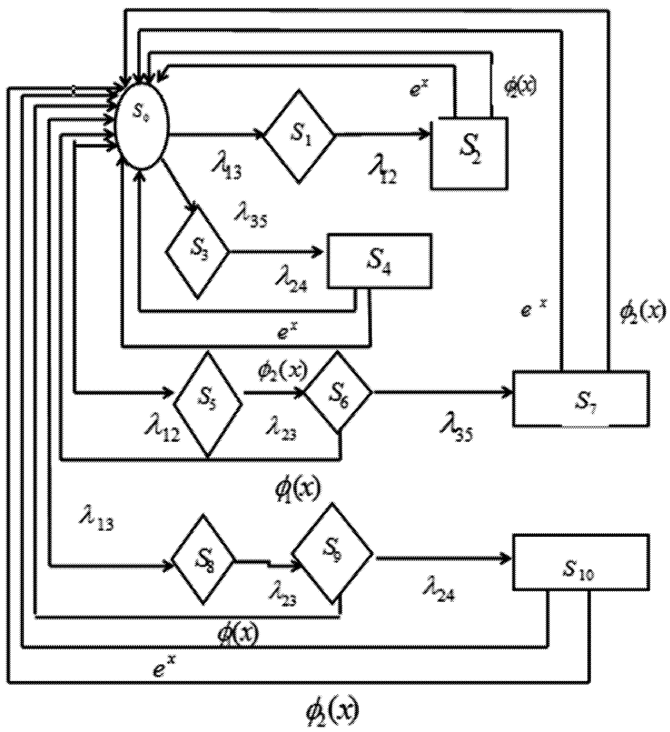

Fig.2: Transition diagram for the acyclic network

2. Solution of the model

Taking Laplace transformation of equations (A.1.1)-(A.1.15) and using equation (A.1.16), we obtain the transition state probabilities of the proposed acyclic network as:

$$\overline{P}_0(s) = \frac{1}{B(s)} \tag{2.1}$$

$$\overline{P}_1(s) = \frac{\lambda_{13}}{(s+\lambda_{12})B(s)} \tag{2.2}$$

$$\overline{P}_2(s) = \frac{\lambda_{13}\lambda_{12}}{(s+\lambda_{12})B(s)}[\frac{(1-\overline{s\varphi(s)}}{s}] \tag{2.3}$$

$$\overline{P}_3(s) = \frac{\lambda_{35}}{(s+\lambda_{24})B(s)} \tag{2.4}$$

$$\overline{P}_4(s) = \frac{\lambda_{24}\lambda_{35}}{(s+\lambda_{24})B(s)}[\frac{(1-\overline{s\varphi(s)}}{s}] \tag{2.5}$$

$$\overline{P}_5(s) = \frac{\lambda_{12}}{(s+\lambda_{23})B(s)} \tag{2.6}$$

$$\overline{P}_6(s) = \frac{\lambda_{23}}{(s+\lambda_{35})B(s)} \tag{2.7}$$

$$\overline{P}_7(s) = \frac{\lambda_{35}\lambda_{12}\lambda_{35}}{(s+\lambda_{12})(s+\lambda_{12})B(s)}[\frac{(1-\overline{s\varphi(s)}}{s}] \tag{2.8}$$

$$\overline{P}_8(s) = \frac{\lambda_{13}}{(s+\lambda_{23})B(s)} \tag{2.9}$$

$$\overline{P}_9(s) = \frac{\lambda_{23}}{(s+\lambda_{24})B(s)} \tag{2.10}$$

$$\overline{P}_{10}(s) = \frac{\lambda_{23}\lambda_{13}\lambda_{24}}{(s+\lambda_{24})(s+\lambda_{23})B(s)}[\frac{(1-\overline{s\varphi(s)}}{s}] \tag{2.11}$$

where

$$B(s) = [(s+c_1) - \frac{\lambda_{13}\lambda_{12}\overline{s\varphi(s)}}{(s+\lambda_{12})} - \frac{\lambda_{24}\lambda_{35}\overline{s\varphi(s)}}{(s+\lambda_{24})} - \frac{\lambda_{23}\lambda_{35}\lambda_{12}\overline{s\varphi(s)}}{(s+\lambda_{12})(s+\lambda_{12})} - \frac{\lambda_{23}\lambda_{35}\lambda_{12}\overline{s\varphi(s)}}{(s+\lambda_{12})(s+\lambda_{12})}]$$

The Laplace transformations of the probabilities that the system is in up (i.e. either good or degraded state) and failed state at any instance are as follows:

$$\overline{P}_{\mathrm{up}}(s) = \overline{P}_0(s) + \overline{P}_1(s) + \overline{P}_3(s) + \overline{P}_5(s) + \overline{P}_6(s) + \overline{P}_8(s) + \overline{P}_9(s) \tag{2.12}$$

$$\overline{P}_{\mathrm{up}}(s) = [1 + \frac{\lambda_{13}}{(s+\lambda_{12})} + \frac{\lambda_{35}}{(s+\lambda_{24})} + \frac{\lambda_{12}}{(s+\lambda_{23})} + \frac{\lambda_{23}}{(s+\lambda_{35})} + \frac{\lambda_{13}}{(s+\lambda_{23})} + \frac{\lambda_{23}}{(s+\lambda_{24})}]\frac{1}{B(s)} \tag{2.13}$$

$$\overline{P}_{\mathrm{down}}(s) = \overline{P}_2(s) + \overline{P}_4(s) + \overline{P}_7(s) + \overline{P}_{10}(s)$$

$$\overline{P}_{\rm down}(s) = \frac{\lambda_{13}\lambda_{12}}{(s+\lambda_{12})B(s)}[\frac{(1-\overline{s\varphi(s)}}{s} + \frac{\lambda_{24}\lambda_{35}}{(s+\lambda_{24})B(s)}[\frac{(1-\overline{s\varphi(s)}}{s}$$
$$+\frac{\lambda_{35}\lambda_{12}\lambda_{35}}{(s+\lambda_{12})(s+\lambda_{12})B(s)}[\frac{(1-\overline{s\varphi(s)}}{s} + \frac{\lambda_{23}\lambda_{13}\lambda_{24}}{(s+\lambda_{24})(s+\lambda_{23})B(s)}[\frac{(1-\overline{s\varphi(s)}}{s}] \quad (2.14)$$

3. Asymptotic Behavior of the network

If $\overline{F}(s)$ is the Laplace transform of F(t), then by Abels lemma, we have

$$lim_{s\to\ 0}s\overline{F}(s) = \ lim_{t\to\infty}F(t) = F$$

Using Abels lemma in equations (2.13) and (2.14), one can obtain the time independent up and down states probabilities as

$$P_{\rm up} = [1+\frac{\lambda_{13}}{\lambda_{12}}+\frac{\lambda_{35}}{\lambda_{24}}+\frac{\lambda_{12}}{\lambda_{23}}+\frac{\lambda_{23}}{\lambda_{35}}+\frac{\lambda_{13}}{\lambda_{23}}+\frac{\lambda_{23}}{\lambda_{24}}]\frac{1}{B(0)} \quad (3.1)$$

$$P_{\rm down} = \lambda_{13}\frac{1}{\varphi}+\lambda_{35}\frac{1}{\varphi}+\lambda_{12}\frac{1}{\varphi}+\lambda_{13}\frac{1}{\varphi} \quad (3.2)$$

where

$$B(0) = lim_{s\to\ 0}B(s)$$

Particular case: When repair follows exponential distribution then, we have

$$\overline{S}_1(s) = \frac{\phi_1(x)}{s+\phi_1(x)}\overline{S}_2(s) = \frac{exp[(x^\theta+(log\phi_2(x))^\theta]^{\frac{1}{\theta}}}{s+exp[(x^\theta+(log\phi_2(x))^\theta]^{\frac{1}{\theta}}} \quad (3.3)$$

4. Numerical computation for the reliability analysis of the network

Using equation (3.3) the network reliability can be computed with the help of equation (2.13). Taking inverse Laplace transformation of equation (2.13) than considering the different failure rates corresponding to different edges in the network $\lambda_{13} = 0.3, \lambda_{12} = 0.2, \lambda_{35} = 0.4, \lambda_{24} = 0.1, \lambda_{23} = 0.15$, repair rates $\phi = 0, \theta = 1$ and $x = 1$ we have

$$Reliability R(t) = P_{\rm up}(t) = e^{1.2t} + 0.3e^{-0.2t} - 0.3e^{-1.2t} + 0.363636e^{-0.1t}$$
$$- 0.363636e^{-1.2t} + 0.19047619e^{-0.15t} - 0.19047619e^{1.2t} + 0.2857142e^{0.15t}$$
$$- 0.2857142e^{1.2t} \quad (4.1)$$

Variation of reliability with respect time of the proposed network has been obtained with the help of equation (4.1). The different values of reliability obtained with respect to time are shown in Table 2 and the corresponding graph has been depicted in Fig 3.

Table2: Reliability vs. Time	
Time	R(t)
0	1
1	0.9424
2	0.8389
3	0.7338
4	0.6367
5	0.5555
6	0.4834
7	0.4212
8	0.3674
9	0.3209
10	0.2806

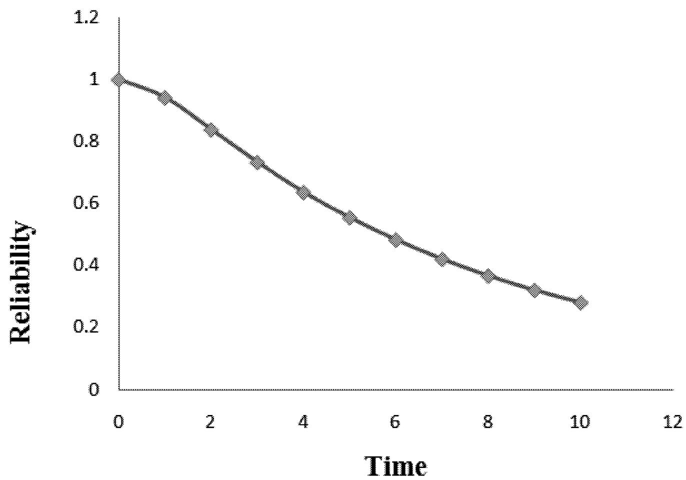

Fig.3: Time vs. Reliability

5. Mean time to failure of the network

Mean time to failure (MTTF) of the network can be obtained by

$$MTTF = lim_{s\to\ 0}\overline{P}_{\mathrm{up}}(s) \tag{5.1}$$

When repair follows exponential distribution one can compute MTTF by using equation (3.3) and (5.1). MTTF of the network for five different cases is discussed as follows:

(i) Setting $\lambda_{13} = 0.3, \lambda_{35} = 0.4, \lambda_{24} = 0.1, \lambda_{23} = 0.15$, and varying$\lambda_{12}$ as$0.01, 0.02, 0.03, 0.04, 0.05$ one can obtain variation of MTTF with respect to λ_{12} from equation (5.1).

(ii) Assuming$\lambda_{12} = 0.2, \lambda_{35} = 0.4, \lambda_{24} = 0.1, \lambda_{23} = 0.15$, and varying$\lambda_{13}$ as$0.01, 0.02, 0.03, 0.04, 0.05$ one can obtain variation of MTTF with respect to λ_{13} from equation (5.1).

(iii) Letting $\lambda_{13} = 0.3, \lambda_{12} = 0.2, \lambda_{35} = 0.4, \lambda_{23} = 0.15$, and varying$\lambda_{24}$

as0.01, 0.02, 0.03, 0.04, 0.05 one can obtain variation of MTTF with respect to λ_{24} from equation (5.1).
(iii) Assuming$\lambda_{13} = 0.3, \lambda_{12} = 0.2, \lambda_{24} = 0.1, \lambda_{23} = 0.15$, and varying$\lambda_{35}$ as0.01, 0.02, 0.03, 0.04, 0.05 one can obtain variation of MTTF with respect to λ_{35} from equation (5.1).
(iv) Setting $\lambda_{13} = 0.3, \lambda_{35} = 0.4, \lambda_{24} = 0.1, \lambda_{12} = 0.2$, and varying$\lambda_{23}$ as0.01, 0.02, 0.03, 0.04, 0.05 one can obtain variation of MTTF with respect to λ_{23} from equation (5.1).
All the MTTFs obtained in the above mentioned cases are listed in Table 3. The variations obtained in MTTF with respect to different failure rates are shown in Figure 4.

Table3: Variations in MTTF w.r.t Failure rates

Failurerate	MTTF w.r.t λ_{12}	MTTF w.r.t λ_{13}	MTTF w.r.t λ_{24}	MTTF w.r.t λ_{35}	MTTF w.r.t λ_{23}
0.01	8.4887	10.4032	38.1943	7.3246	47.0814
0.02	8.46404	10.2604	21.5277	7.3576	26.249
0.03	8.4466	10.1262	15.9721	7.3856	23.16666
0.04	8.42948	9.9999	13.1941	7.4202	15.8327
0.05	8.41269	9.8809	11.5278	7.4509	13.7495

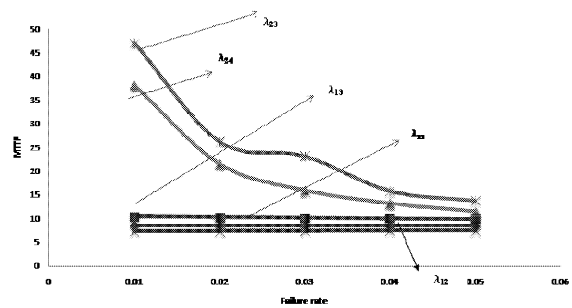

Fig.4: Failure rate vs. MTTF

6. Sensitivity of the network

Sensitivity of the network reliability with respect to any parameters ξ can be obtained by
$S_{\xi} = \frac{\partial R(t)}{\partial \xi}, where \xi = \lambda_{12}, \lambda_{13}, \lambda_{24}, \lambda_{35} and \lambda_{23}$

Now using equation (6.1) and (2.13), we can have expressions for the sensitivity of the proposed network with respect to failure parameters λ_{12} ,λ_{13} ,λ_{24},λ_{35} and λ_{23}.Inverse Laplace transform of developed expressions with respect to different parameters are as follow:

$$\frac{\partial R(t)}{\partial \lambda_{12}} = \frac{3}{10}e^{-t(\lambda_{12}+1)} - \frac{3}{10}e^{-(\lambda_{12}t)} + \frac{3}{10}e^{-t(\lambda_{12}+1)} + \frac{40}{(10\lambda_{12}+9)^2}e^{-t(\lambda_{12}+1)} - \frac{40}{(10\lambda_{12}+9)^2}e^{\frac{-t}{10}}$$
$$+\frac{4}{(10\lambda_{12}+9)}te^{-t(\lambda_{12}+1)} + \frac{15}{(5\lambda_{12}+3)^2}e^{-t(\lambda_{12}+1)} - \frac{15}{(5\lambda_{12}+3)^2}e^{\frac{-2t}{5}} - \frac{15}{(10\lambda_{12}+9)^2}e^{\frac{-t}{10}}$$
$$+\frac{3}{(20\lambda_{12}+12)}e^{-t(\lambda_{12}+1)} + \frac{6}{(20\lambda_{12}+17)}e^{-t(\lambda_{12}+1)} + \frac{3}{(20\lambda_{12}+18)}e^{-t(\lambda_{12}+1)} + \frac{120}{(20\lambda_{12}+17)^2}e^{-t(\lambda_{12}+1)}$$
$$+\frac{120}{(20\lambda_{12}+17)^2}e^{\frac{-3t}{20}} + \frac{15}{(10\lambda_{12}+9)^2}e^{-t(\lambda_{12}+1)} + \frac{400}{(20\lambda_{12}+17)^2}\lambda_{12}e^{-t(\lambda_{12}+1)} - \frac{400}{(20\lambda_{12}+17)^2}\lambda_{12}e^{\frac{-3t}{20}}$$
$$+ \frac{20}{(20\lambda_{12}+17)}\lambda_{12}e^{-t(\lambda_{12}+1)} \quad (6.2)$$

$$\frac{\partial R(t)}{\partial \lambda_{24}} = \frac{10}{(5\lambda_{24}-6)^2}e^{-(\lambda_{24}t)} + \frac{15}{(5\lambda_{24}-6)^2}e^{-(\lambda_{24}t)} - \frac{15}{(5\lambda_{24}-6)^2}e^{\frac{-6t}{5}} + \frac{3}{(20\lambda_{24}-24)}te^{(\lambda_{24}t)}$$
$$+ \frac{2}{(5\lambda_{24}-6)}te^{(\lambda_{24}t)} - \frac{10}{(5\lambda_{24}-6)^2}e^{\frac{-6t}{5}} \quad (6.3)$$

$$\frac{\partial R(t)}{\partial \lambda_{35}} = \frac{10}{(10\lambda_{35}+7)}e^{\frac{-t}{10}} + \frac{15}{64}e^{-(\lambda_{35}t)} - \frac{3}{16}te^{(\lambda_{24}t)} - \frac{t}{5}e^{-t(5\lambda_{35}+4)} + \frac{t}{5}e^{-t(5\lambda_{35}+4)}$$
$$+ \frac{t}{5}e^{-t(5\lambda_{35}+4)} - \frac{10}{(10\lambda_{35}+7)}e^{\frac{-t(5\lambda_{35}+4)}{5}} + \frac{15}{64}e^{\frac{-t(5\lambda_{35}+4)}{5}} - \frac{3}{10}e^{\frac{-t(5\lambda_{35}+4)}{5}}$$
$$+\frac{t}{25}e^{\frac{-t(5\lambda_{35}+4)}{5}} + e^{\frac{-t(5\lambda_{35}+4)}{5}} + \frac{t}{5}e^{\frac{-t(5\lambda_{35}+4)}{5}} - \frac{3}{20}e^{\frac{-t(5\lambda_{35}+4)}{5}} - \frac{3}{5}e^{\frac{-t(5\lambda_{35}+4)}{5}} + \frac{3}{25}e^{\frac{-t(5\lambda_{35}+4)}{5}} - \lambda_{35}e^{\frac{-t(5\lambda_{35}+4)}{5}}$$
(6.4

$$\frac{\partial R(t)}{\partial \lambda_{13}} = \frac{5}{(10\lambda_{13}+2)}e^{\frac{-t}{5}} + \frac{20}{(40\lambda_{13}+9)}e^{\frac{-3t}{20}} - \frac{5}{(10\lambda_{13}+2)}e^{\frac{-t(10\lambda_{13}+3)}{5}} -$$
$$\frac{20}{(40\lambda_{13}+9)}e^{\frac{-t(10\lambda_{13}+3)}{5}} - \frac{t}{5}e^{\frac{-t(10\lambda_{13}+3)}{5}} - \frac{3}{(5(4\lambda_{13}+1)^2})e^{\frac{-t}{10}} + \frac{3}{(5(4\lambda_{13}+1)^2})e^{\frac{-t(10\lambda_{13}+3)}{5}}$$
$$-\frac{8}{(5(4\lambda_{13}+1)^2})e^{\frac{-t}{10}} + \frac{8}{(5(4\lambda_{13}+1)^2})e^{\frac{-t(10\lambda_{13}+3)}{5}} - \frac{80}{(4\lambda_{13}+9)^2}e^{\frac{-3t}{20}} + \frac{80}{(4\lambda_{13}+9)^2}e^{\frac{-t(10\lambda_{13}+3)}{5}}$$
$$-\frac{15}{(4(10\lambda_{13}+1)^2})e^{\frac{-2t}{5}} + \frac{15}{(4(10\lambda_{13}+1)^2})e^{\frac{-t(10\lambda_{13}+3)}{5}} + \frac{25\lambda_{13}}{(4(5\lambda_{13}+1)^2})e^{\frac{-t(10\lambda_{13}+3)}{5}}$$
$$-\frac{25\lambda_{13}}{(4(5\lambda_{13}+1)^2})e^{\frac{-t}{5}} + \frac{5\lambda_{13}t}{(10\lambda_{13}+2)}e^{\frac{-t(10\lambda_{13}+3)}{5}} + \frac{400\lambda_{13}}{(40\lambda_{13}+9)}e^{\frac{-t(10\lambda_{13}+3)}{5}} - \frac{400\lambda_{13}}{(40\lambda_{13}+9)}e^{\frac{-3t}{20}}$$
$$+\frac{20\lambda_{13}t}{(40\lambda_{13}+9)}e^{\frac{-t(10\lambda_{13}+3)}{5}} + \frac{4t}{(40\lambda_{13}+9)}e^{\frac{-t(10\lambda_{13}+3)}{5}} - \frac{3t}{(40\lambda_{13}+4)}e^{\frac{-t(10\lambda_{13}+3)}{5}}$$
(6.5)

The computed sensitivities with respect to different parameters are listed in Table 4. The same is also depicted in Figure 5.

Table4: Sensitivities w.r.t. different parameters					
Time	$S\lambda_{12}$	$S\lambda_{13}$	$S\lambda_{35}$	$S\lambda_{23}$	$S\lambda_{24}$
0	0.00000	0.00000	0.00000	0.00000	0.00000
1	-5.41994	2.946685	-1.82551	-0.33854	-0.29901
2	-23.5437	11.96128	-10.6537	-0.30977	-0.23265
3	-78.4001	39.73755	-38.5026	-0.17368	-0.03419
4	-235.319	119.7443	-118.641	-0.03886	0.188641
5	-667.298	340.4482	-339.479	0.05726	0.393291
6	-1824.47	932.0885	-931.24	0.106631	0.5662
7	-4862.35	2485.818	-2485.08	0.111935	0.705562
8	-12715.2	6502.973	-6502.32	0.078946	0.813897
9	-32767.9	16762.53	-16762	0.013617	0.895022
10	-83469.5	42705.8	-42705.3	-0.0789	0.952881

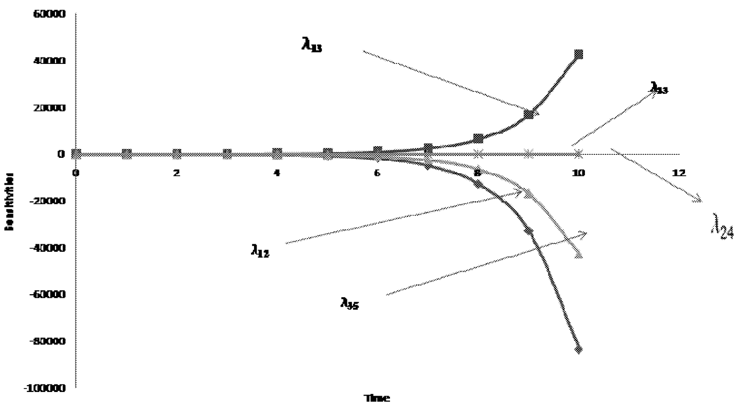

Fig.5: Sensitivity vs. Time

7. Conclusion

In the present study, various reliability measures have been computed for the acyclic transmission network with the help of the minimal cuts and the Markov processes incorporating different types of failure. In this model various reliability measures like transition probabilities, asymptotic behavior, reliability, MTTF and sensitivities with respect to different parameters have been obtained with the help of the proposed method unlike done in the past. Figure 3 provides the variation of reliability with respect to time. By observing the figure one can visualize that it decreases from its initial stage

with respect to time.
From Figure 4, we can easily see that MTTF with respect toλ_{12},λ_{13} *and* λ_{35} are continuously and slowly decreasing though, the variation is found to be decreasing linearly. Further, it is observed that the MTTF with respect toλ_{24} is decreasing exponentially. The highest and the lowest value of MTTF have been found with respect to failure rates of edgesλ_{23} and λ_{35} respectively.
Figure 5 shows the sensitivity of the proposed network with respect to different parameters. Critical examination of the figure reveals that sensitivity of system corresponding to the parametersλ_{12}andλ_{35} is decreasing with respect to time whereas it is increasing corresponding to parameterλ_{13} . Moreover, it is observed that sensitivity of the system corresponding to the parameters $\lambda_{23}$and λ_{24} firstly decreases and then slowly increases with respect to time. The system is found to be most sensitive with respect to parameterλ_{13}.

References

[1] Colbourn, C.J., 1987. The combinatorics of network reliability. Oxford University Press, Inc..

[2] Gertsbakh, I. and Shpungin, Y., 2011. Network reliability and resilience. Springer Science and Business Media.

[3] Gertsbakh, I.B. and Shpungin, Y., 2016. Models of network reliability: analysis, combinatorics, and Monte Carlo. CRC press.

[4] Kumar, D. and Singh, S.B., 2013. Reliability Analysis of Embedded System with Different Modes of Failure Emphasizing Reboot Delay. International Journal of Applied Science and Engineering, 11(4), pp.449-470.

[5] Levitin, G., 2001. Reliability evaluation for acyclic consecutively connected networks with multistate elements. Reliability Engineering and System Safety, 73(2), pp.137-143.

[6] Levitin, G., 2003. Reliability evaluation for acyclic transmission networks of multi-state elements with delays. IEEE Transactions on Reliability, 52(2), pp.231-237.

[7] Levitin, G., 2005. The universal generating function in reliability analysis and optimization (Vol. 6). London: Springer.

[8] Malinowski, J. and Preuss, W., 1996. Reliability evaluation for tree-structured systems with multistate components. Microelectronics Reliability, 36(1), pp.9-17.

[9] Munjal, A. and Singh, S.B., 2014. Reliability analysis of a complex repairable system composed of two 2-out-of-3: G subsystems connected in parallel. Journal of reliability and statistical studies, pp.89-111.

[10] Nailwal, B. and Singh, S.B., 2011. Performance evaluation and reliability analysis of a complex system with three possibilities in repair with the application of copula. International Journal of Reliability and Applications, 12(1), pp.15-39.

[11] Nailwal, B. and Singh, S.B., 2012. Reliability measures and sensitivity analysis of a Complex matrix system including power failure. International Journal of Engineering-Transactions A: Basics, 25(2), p.115.

[12] Nailwal, B. and Singh, S.B., 2016. Reliability Analysis of Two Dissimilar-Cold Standby Redundant Systems Subject to Inspection with Preventive Maintenance Using Copula. Mathematical Concepts and Applications in Mechanical Engineering and Mechatronics, p.201.

[13] Nelsen, R.B., 2006. An introduction to copulas, 2nd. New York: Springer Science Business Media.

[14] Ram, M. and Singh, S.B., 2008. Availability and cost analysis of a parallel redundant complex system with two types of failure under preemptive-resume repair discipline using Gumbel-Hougaard family copula in repair. International Journal of Reliability, Quality and Safety Engineering, 15(04), pp.341-365.

[15] Srinivasan, S.K. and Subramanian, R., 2006. Reliability analysis of a three unit warm standby redundant system with repair. Annals of Operations Research, 143(1), p.227-235.
[16] Yeh, W.C., 2003. Multistate-node acyclic networks reliability evaluation based on MC. Reliability Engineering and System Safety, 81(2), pp.225-231.
[17] Yeh, W.C., 2006. The k-out-of-n acyclic multistate-node networks reliability evaluation using the universal generating function method. Reliability Engineering and System Safety, 91(7), pp.800-808.
[18] Yeh, W.C., 2012. A modified universal generating function algorithm for the acyclic binary-state network reliability. IEEE Transactions on Reliability, 61(3), pp.702-709.

Appendix A.1

With the help of state transition diagram, we may obtain the difference equations for state S0 by
$P_0(t+\Delta t) = [1-(\lambda_{13}\Delta t+\lambda_{35}\Delta t+\lambda_{12}\Delta t+\lambda_{13}\Delta t)P_0(t)] + \int_0^\infty \varphi(x)\ P_2(x,t)dx + \int_0^\infty \varphi(x)\ P_4(x,t)dx + \int_0^\infty \varphi(x)\ P_7(x,t)dx + \int_0^\infty \varphi(x)\ P_{10}(x,t)dx$
Other difference equations can be obtained the same way as mentioned above with the help of transition diagram. When $\Delta t \to 0$, we can have the differential equations corresponding to different states.

Formulation of Mathematical model

By probability considerations and continuity arguments we can obtain the following set of difference-differential equations governing the present mathematical model.
$[\frac{d}{dt}+\lambda_{13}+\lambda_{35}+\lambda_{12}+\lambda_{13}]P_0(t) = \int_0^\infty \varphi(x)\ P_2(x,t)dx + \int_0^\infty \varphi(x)\ P_4(x,t)dx + \int_0^\infty \varphi(x)\ P_7(x,t)dx + \int_0^\infty \varphi(x)\ P_{10}(x,t)dx$
$[\frac{d}{dt}+\lambda_{12}]P_1(t) = \lambda_{13}P_0(t)$
$[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\varphi(x)]P_2(x,t) = 0$
$[\frac{d}{dt}+\lambda_{24}]P_3(t) = \lambda_{35}P_0(t)$
$[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\varphi(x)]P_4(x,t) = 0$
$[\frac{d}{dt}+\lambda_{23}]P_5(t) = \lambda_{12}P_0(t)$
$[\frac{d}{dt}+\lambda_{35}]P_6(t) = \lambda_{23}P_0(t)$
$[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\varphi(x)]P_7(x,t) = 0$
$[\frac{d}{dt}+\lambda_{23}]P_8(t) = \lambda_{13}P_0(t)$
$[\frac{d}{dt}+\lambda_{24}]P_9(t) = \lambda_{23}P_0(t)$
$[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\varphi(x)]P_{10}(x,t) = 0$
Boundary conditions $P_2(0,t) = \lambda_{12}P_1(t)$
$P_4(0,t) = \lambda_{24}P_3(t)$
$P_7(0,t) = \lambda_{35}P_6(t)$
$P_{10}(0,t) = \lambda_{24}P_9(t)$
Initial conditions

$$P_{\mathrm{i}}(0) = \begin{cases} 1, & \text{if } i = 0; \\ 0 & \text{if } i > 1 \end{cases}$$

Department of Mathematics, Statistics and Computer Science,, G.B. Pant University of Agriculture and Technology, Pantnagar, India
Email address: bsdipi1992@gmail.com

Department of Mathematics, Statistics and Computer Science,, G.B. Pant University of Agriculture and Technology, Pantnagar, India
Email address: drsurajbsingh@yahoo.com